Calibrating a non-invasive cosmic ray soil moisture probe for snow water equivalent

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Abstract

I developed a function for converting neutron counts from a Hydroinnova cosmic-ray soil moisture sensor (CRS) to snow water equivalent. The function requires that the counting rate first be corrected, and that the probe is also calibrated for soil moisture, or that the soil moisture conditions are otherwise known. The function has the form

$$SWE = -\Lambda \ln\left(\frac{N - N_{\mathrm{wat}}}{N_0 - N_{\mathrm{wat}}}\right)$$

where Λ is an attenuation length, and N_{wat} is the counting rate over deep (> 30 cm) water. From neutron transport simulations Λ was found to have a value of 4.8 cm, and N_{wat} was found to be 0.24 N_0, where N_0 is the counting rate over dry soil. A practical limit to the technique is probably reached at 3Λ. Field data from a snow pillow validate the function by showing that it displays a reasonable curvature, and implies a fairly reasonable water content of ~0.25 kg kg^{-1} for the study site. While the maximum SWE that can be measured cannot be much greater than several times Λ, data from a mountain site in New Mexico demonstrate that CRS data can still be valuable even when the SWE locally exceeds the nominal limits of the technique.

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Contents

1. Introduction

While better known for its ability to monitor soil moisture, Hydroinnova's Cosmic Ray Sensor (CRS; Figure 1) can also be used to measure snow water equivalent (SWE) over the same uniquely large footprint as in moisture determinations. Advantages of the technique, in addition to the big footprint, are that it is non invasive (sensor located above the snow pack), automatic, easy to install and requires little maintenance.

Figure 1. Dual channel CRS probe installed near Flagstaff, AZ.

The basis of the technique is that snow blocks neutrons from coming out of the ground (Figure 2). The neutrons originate naturally through cosmic ray interactions in the air and ground, and to a small extent the snow pack itself. The relationship is one of mass attenuation, such that the amount of attenuation is directly related to the mass of intervening snow, and by extension the amount of SWE.

With the spread of CRS technology into colder climates, the relationship between above-snow neutron intensity and SWE is receiving more attention. The equipment used for SWE monitoring is in this case identical to that for soil moisture determinations, and when it comes to installation, there are few additional considerations, other than being mindful of not allowing the sensor or solar panel to become buried in snow. The goal of this document is to provide a more rigorous foundation for converting raw CRS data to SWE.

As with all SWE monitoring methods, the CRS technique has its limitations. Most notable of these are the temporal resolution (integration time for counts), the amount of SWE that can be measured, and the ability to distinguish SWE from soil moisture. However, as I show in an example, even at sites where SWE locally exceeds the upper limit of the technique, the larger observation scale provided by the CRS can provide valuable information on a heterogeneous snowpack. Furthermore, CRS probes are often installed with soil moisture first in mind, and SWE is simply a valuable supplemental data set that can be extracted from the CRS time series. Such data can be illuminating even if the SWE limit is reached at the site, because information about the timing of early season snow events and spring melt are still obtained where SWE data are otherwise lacking.

When observations of deep SWE are needed, other sensor options are available, including Hydroinnova's SnowFox system. The SnowFox gives a point measurement and can be calibrated up to several meters of SWE—and theoretically even more. This paper will concentrate on obtaining SWE from the CRS system, although some principles, for example relating to atmospheric pressure and solar corrections, apply to both methods.

2. Converting raw data to corrected data

The primary data recorded by the CRS is the neutron counting rate above the snow pack. The basis of the technique is that any deviations from the baseline counting rate is inversely proportional to the amount of soil moisture and/or SWE, the two variables of primary interest. However there are several other environmental variables that can influence the baseline neutron intensity, and cause fluctuations in the raw counting rate, N_{raw}. These unwanted fluctuations are eliminated by applying a correction factor $F(t)$, such that

$$N = N_{raw} F(t) \quad (1)$$

The correction factor $F(t)$ can be decomposed into individual correction factors for the processes under consideration. Currently the main correction factors account for barometric pressure (f_{bar}), solar activity (f_{sol}) and atmospheric humidity (f_{hum}). The total correction factor is then

$$F(t) = f_{bar} \cdot f_{sol} \cdot f_{hum} \quad (2)$$

The first of these multipliers is the barometric pressure correction factor, calculated as

$$f_{bar} = \exp\left[\beta\left(p(t) - p_0\right)\right] \quad (3)$$

where $p(t)$ is the barometric pressure recorded at the site in hPa, p_0 is a fixed reference pressure (also in hPa), usually taken to be an approximate long term average for the site, or calculated from the elevation and a model representation of the atmosphere. β is the pressure coefficient, which at high to mid latitude can be assumed to be 0.0077 hPa^{-1}.

The second factor corrects for variations in solar activity, and is calculated as

$$f_{sol} = \frac{M_0}{M(t)} \quad (4)$$

where $M(t)$ is the counting rate of a neutron monitor at time t, and M_0 is the counting rate at an arbitrarily chosen reference time. The factor f_{sol} is calculated on an hourly basis from the Jungfraujoch neutron monitor and reported on a Hydroinnova maintained web portal, http://nearfld.com/reguser/solar/. Procedures exist to scale this factor more accurately to other locations, particularly low latitude locations, and one is encouraged to contact the author for the latest information. The third factor, f_{hum}, adjusts the counting rate for changes in the absolute humidity of the atmosphere (H). As absolute humidity rises the counting rate tends to drop. According to Rosolem et al. (2013), the neutron rate can be corrected with the formula

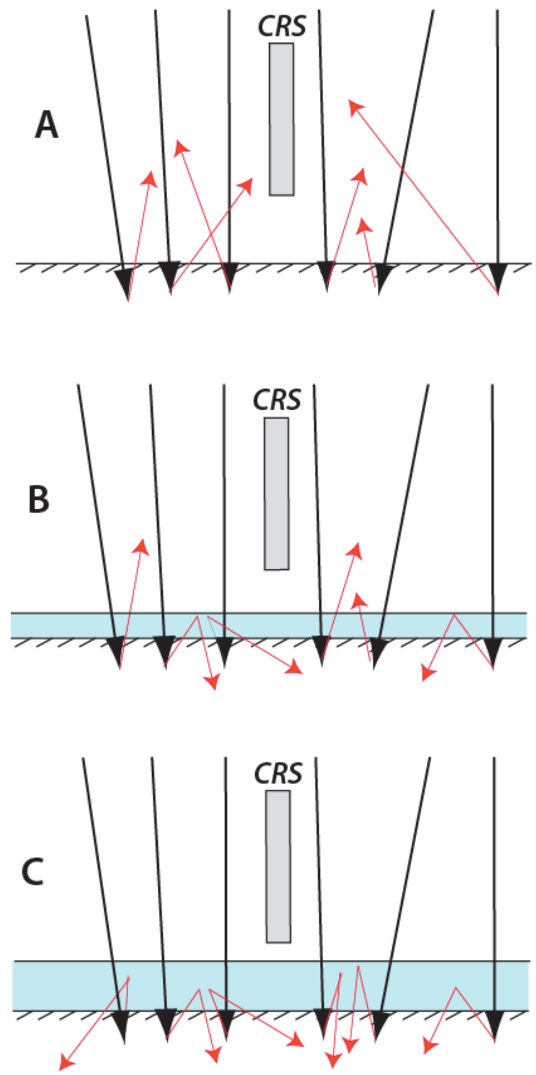

Figure 2. Influence of SWE on cosmic ray neutrons: (A) many neutrons produced in ground escape to atmosphere; (B) some are blocked by snow; and (C) nearly all are blocked by snow.

$$f_{hum} = 1 + 0.0054 \cdot H(t) \quad (5)$$

where $H(t)$ is in units of g m^{-3}. The absolute humidity of a well mixed atmosphere can be calculated from a local measurement of air temperature (T) and relative humidity (U) by first calculating the saturation vapour pressure,

$$e_w = 6.112 \exp\left(\frac{17.62T}{243.12+T}\right) \tag{6}$$

where e_w is in hPa and T is in °C (WMO Guide, 2008), and then calculating absolute humidity as

$$H = \frac{U}{100}\left(\frac{e_w k}{T+273.16}\right) \tag{7}$$

where U is expressed as a percentage and k is a constant equal to 216.68 g k J^{-1} (Parish and Putnam, 1977).

The idea behind the correction factors is to normalize the counting rate to a set of reference conditions; the reference barometric pressure, reference humidity, and reference solar activity level. The exact reference levels chosen are not important as long as the user is consistent, and normalizes N_0 (see next section) to exactly the same conditions.

3. Theoretical calibration function

The corrected counting rate N will tend to decrease as SWE increases, as shown in Figure 2. However, Figure 2 is not a complete picture of the processes at work, since it does not show the generation of neutrons in the air and water. Because of these neutrons, the counting rate will not decrease to zero; at some point it will reach a constant value that is the counting rate over an infinite depth of water, N_{wat}. The counting rate over water represents contributions from neutrons that are produced in the atmosphere and are then reflected off of water, or are produced directly in the water and escape to the atmosphere. As the thickness of water decreases, the counting rate becomes increasingly influenced by neutrons that are produced in the ground and escape upward through the snowpack. Conversely, when the thickness of water is large, the counting rate N approaches N_{wat}.

The reason for taking the difference N-N_{wat} is to isolate the contribution of neutrons that ultimately originate from or with the ground. It seems reasonable to assume that, as with other types of radiation, the attenuation of these neutrons by water follows a linear law, such that

$$\frac{dN}{dSWE} = -\frac{N - N_{wat}}{\Lambda} \tag{8}$$

This first order differential equation can be solved as follows:

$$\int \frac{1}{N\text{-}N_{wat}}\, dN = -\frac{1}{\Lambda}\int dSWE \tag{9}$$

$$\ln(N - N_{wat}) = -\frac{SWE}{\Lambda} + C \tag{10}$$

$$N - N_{wat} = \exp\left(-\frac{SWE}{\Lambda}\right)\exp(C) \tag{11}$$

When SWE=0, N is simply the counting rate over soil. Because this zero-snow counting rate is principally determined by the soil water content, θ, I refer to it is N_θ to emphasize the dependence on water content. Hence, the initial condition is that at SWE=0, N=N_θ. Solving for the integration constant,

$$C = \ln(N - N_{\mathrm{wat}}) \tag{12}$$

and then substituting this constant back into equation 11 gives

$$N = (N_\theta - N_{\mathrm{wat}}) \exp\left(-\frac{SWE}{\Lambda}\right) + N_{\mathrm{wat}} \tag{13}$$

This equation is advantageous in that the dependence on SWE requires only two small parameters, both of which are physically meaningful. Another advantage is that it is easily inverted to give SWE as a function of N:

$$SWE = -\Lambda \ln\left(\frac{N - N_{\mathrm{wat}}}{N_\theta - N_{\mathrm{wat}}}\right) \tag{14}$$

Recommended parameter values are:

$$\Lambda = 4.8 \text{ cm}$$
$$N_{\mathrm{wat}} = 0.24 N_0$$

Where N_0 is the theoretical counting rate over dry soil. It is theoretical because it is usually calculated through a calibration function, and not measured directly. These parameter values were obtained from neutron transport simulations as described in Section 5, and validated using independent SWE data from a site in New Mexico. Error propagation based on the neutron counting rate is described in the next section.

The value of N_0 is found by rearranging the soil moisture calibration function (Desilets et al., 2010):

$$N_0 = \frac{F(t) N_\theta}{\dfrac{a_0}{\theta_g \rho_{bd} + a_2} + a_1} \tag{15}$$

$$a_0 = 0.0808$$
$$a_1 = 0.3720$$
$$a_2 = 0.1150$$

The gravimetric water content, θ_g, and soil bulk density, ρ_{bd}, are usually found through local sampling, for example with a soil coring apparatus.

Importantly, the formula above work only for the moderated CRS channel, for which intensity decreases monotonically with SWE. This is because the addition of small amounts of SWE (up to ~3 cm) to bare ground tends to increase the thermal neutron intensity, a behavior which is quite opposite to the assumed linear dependence. However, for larger amounts of SWE thermal neutrons come into equilibrium with epithermal neutrons, and the dependency is the same for the two components. This idiosyncracy bears mentioning for two reasons. First, the moderated (primary) channel of Hydroinnova's CRS probe is known to have a small thermal neutron contribution (and more significant subcadmium contribution). However, the total response, nonetheless, appears to be monotonic with SWE and, therefore similar to the function for epithermal neutrons. The theoretical calibration equation therefore works well. Second, CRS moderated probes are often accompanied by a bare channel, which is primarily sensitive to thermal neutrons. The use of the bare channel in SWE measurements remains the topic of ongoing research. This paper deals only with the moderated channel.

Lastly, it should be emphasized that this calibration function is only an approximation, and can possibly be improved, for example by adding other terms to the equation. The function presented above is a good foundation on which to build a higher order calibration function.

4. Error propagation

Errors can be propagated through the calibration function by utilizing the general error propagation formula

$$\sigma_u^2 = \left(\frac{\partial u}{\partial x}\right)^2 \sigma_x^2 \tag{16}$$

for a continuous function $u=f(x)$. Written for SWE:

$$\sigma_{SWE}^2 = \left(\frac{\partial SWE}{\partial N}\right)^2 \sigma_N^2 \tag{17}$$

and from equation 8 it is known that:

$$\frac{dN}{dSWE} = -\frac{N - N_{wat}}{\Lambda}$$

The uncertainty in N is dominated by the statistical counting uncertainty, which is given by

$$\sigma_N = \sqrt{N} \tag{18}$$

Putting this all together gives

$$\sigma_{SWE} = \frac{\Lambda}{N - N_{wat}} \sqrt{N} \tag{19}$$

Importantly, the value of N should always reflect the total number of counts over the time interval of interest. It is essential to do this, particularly when data are aggregated in order to improve counting statistics (e.g., moving averages). For example, if the average hourly counting rate is 1000 counts per hour, and one takes a 12 hr average of the counting rate in order to smooth the data, then for the purpose of error propagation N = 12,000 counts per 12 hours. Similarly, N_{wat} would also be multiplied by 12 in this case, to give the same units of counts per 12 hours. Failure to use a reasonable a time window, or failure to maintain a consistent time base for N and N_{wat} will result in error estimates that are unrealistically large.

5. Parameters for the calibration function

The SWE calibration function has only three parameters, N_θ, N_{wat} and Λ. Since N_θ represents the counting rate immediately prior to the first snowfall, it will usually be known beforehand (unless the instrument is installed with snow already on the ground). In principal N_{wat} can be determined by taking measurements over water, or from data collected over deep snow (SWE > 30 cm), although this work relies on neutron transport simulations. The most difficult parameter to constrain is Λ, which is the exponential slope of the calibration function. I determined Λ from neutron transport simulations performed for several different soil water contents, as discussed below.

Monte Carlo simulations

I used the Monte Carlo N-Particle eXtended (MCNPX) code to simulate the hadronic cascade and its interactions with the atmosphere, snow and soil (MCNPX, 2011). Our source consisted of atmospheric secondary neutrons with energies ranging from 0.1 to 12.8 GeV. The source probability was partitioned according to a seven bin histogram following the atmospheric neutron spectrum in Hughes and Marsden (1966). This source was distributed along a plane placed in the atmosphere 345 g cm^{-2} above the ground, a distance thought sufficient enough to allow low energy neutrons to come into equilibrium with the energetic source spectrum, thus closely approximating the spectrum that would be observed in a full 1033 g cm^{-2} thick column of atmosphere. The specific humidity of the atmosphere was 10 g kg^{-1}. The ground was modeled as a 560 g cm^{-2} thick slab of SiO_2 having a variable water content. The snow pack was modeled as a continuous layer of water between the ground and atmosphere. The CRS probe response was simulated by allowing the contribution of counts below 0.5 eV to be 25% based on the work of Andreasen et al. (2016).

The best fit value for Λ was found by minimizing the absolute deviations in SWE calculated over the SWE range 0 to 10 cm. This procedure yielded Λ = 4.8 cm. The simulations revealed that N_{wat}=0.24N_0. With these parameter values our equation reproduces the simulated SWE to within 1 cm over a range of SWE from 0 to 10 cm and for soil water contents from 0.07 to 0.36 kg kg^{-1}.

6. Validation

A CRS probe has been deployed for several years in the Jémez Mountains of northern New Mexico, USA (35.8895°N, -106.5319°E). The site is a mixed conifer forest at an elevation of 3038 m (Figure 4), in a region known for its dry and sunny climate—although winter snow fall can be significant at this elevation, with a spring snowpack often exceeding 20 cm of SWE. As a form of validation, the neutron counting rate is compared to SWE recorded by a snow pillow approximately 10 m distant. The observed counting rate (Figure 5) shows a dependence on SWE that is consistent with the calibration function derived here if one assumes a soil water content of 0.25 kg kg^{-1}, which seems reasonable for this site.

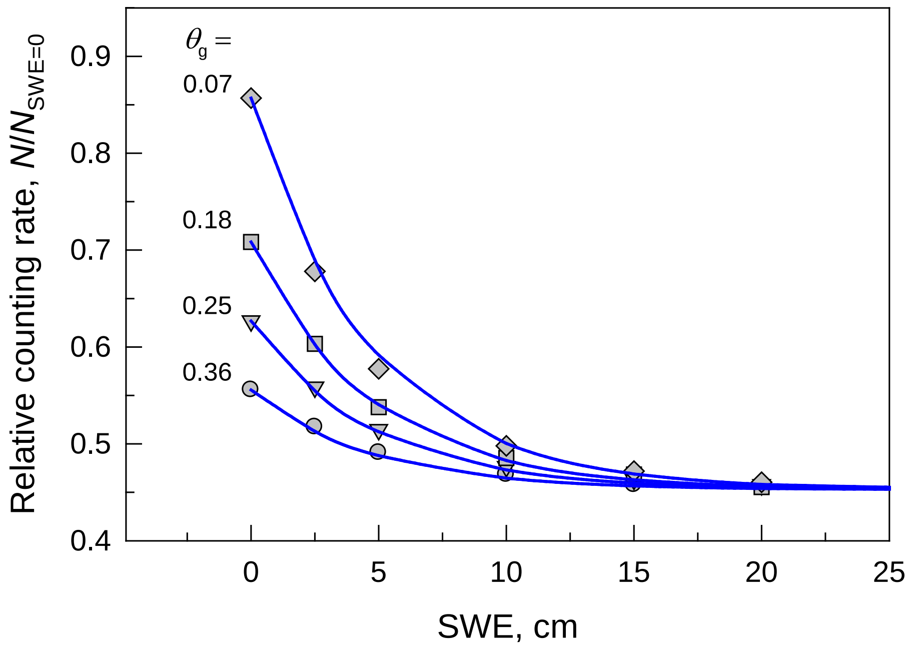

Figure 3. Calibration function (solid lines) fitted to MCNPX results for (symbols) for varying θ_g.

Figure 4. Field site in the Jémez Mountains, NM. In November, 2009 the CRS probe was relocated from a inside of the trailer to pole installed a few meters to the right of the snow pillow.

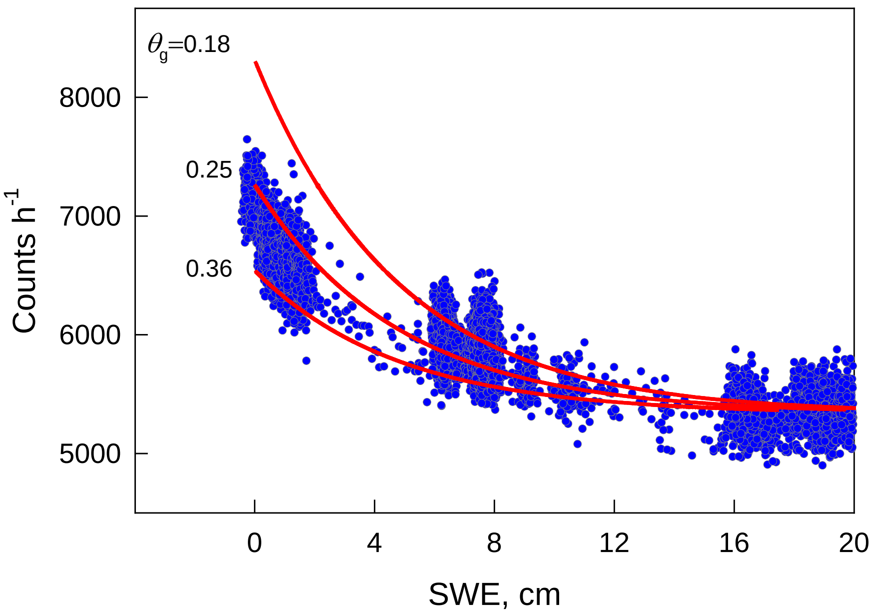

Figure 5. Counting rate from a CRS1000 plotted against SWE from a snow pillow several meters away (open circles). The site is in the Jémez Mountains of New Mexico and measurements are during water year 2010. The lines are calibration curves calculated for different water contents.

7. Field example

To demonstrate how data from a CRS probe can be illuminating even in an area that regularly pushes the upper SWE limit of the technique, I show CRS data from the winter of 2008-2009 and compare the estimated SWE to results from a snow pillow located 10 meters away. The field site is the same one in the Jèmez Mountains, but during this period the CRS probe was located in the trailer shown in Figure 4.

A few features of the comparative SWE time series (Figure 6) are immediately apparent. First, the data diverge widely after the first two snow events, with the CRS probe measuring far less accumulated SWE than the snow pillow. Second, the snowpack measured by the CRS probe is, throughout the winter, more dynamic than the one seen by the snow pillow. (An exception is from mid March to mid April, but the high frequency oscillations in the snow pillow are believed to an instrumental anomaly.) This higher level of dynamism for the CRS probe is consistent with the lower overall SWE level implied by the CRS data, since a significant part of the dynamics would in fact be the rapid disappearance of the snow through sublimation and melting, leading to less snow overall. In other words, the more dynamic the snow pack, the less of it there tends to be.

High levels of sublimation have independently been measured from an eddy covariance tower at this site, located approximately 95 m from the CRS probes. Molotch et. al. (2007) observed a rate as high as 3.7 mm d^{-1} from the canopy at this site. Such a high rate is consistent with the SWE dynamics witnessed with the CRS probe, although some of the more extreme dynamics in the CRS data set would require at least temporarily higher rates, or melting in addition to sublimation.

Third, several minor snow events are evident in the CRS data from mid January to March but are absent from the snow pillow time series. It could be that the response of the snow pillow is slower in mid winter (due perhaps to structure of the snow pack) and that snow from the most recent storm disappeared before the pillow has had time to respond. It could also be that bare patches of ground have opened up between storms, and that small snow events which cover these patches cause a disproportionate (non linear) response in the CRS, which sees a much wider area than the snow pillow. Part of the explanation could also be related to the tendency for the CRS probe to see canopy intercepted snow as part of the snow pack, whereas the snow pillow clearly does not.

Fourth, several events trigger a significant and fairly proportionate SWE response in both the pillow and the CRS probe over the same mid January to March time frame. Most notable are the events begining on January 1, March 12, and April 11. Such events suggest that the response of the CRS probe is still fairly linear, which means that the influence of the non-linear calibration curve, combined with heterogenous snow cover, is not much of an obstacle to interpreting this data set. A heterogeneous snow cover, particularly from February on, is typical at this site (Figure 7).

Fifth, an important characteristic of the 2009 water year snowpack is the long period of stability from January through April, bracketed on one end by a small number of major early season snow events and on the other end by the spring melt. What is interesting is that wide area snow pack, as measured by the CRS, seems to reach a fairly stable plateau at 5 cm despite the significant exposure to sublimation, a trait that is demonstrated by the rapid disappearance of SWE following most of the snow events that occur during the plateau months. This may suggest something about how snow tends to persist in favored areas at this site (for example, at the snow pillow) and how it rapidly disappears from unfavored areas. Furthermore the CRS data may give insights into how the stabilization of the early season snow pack lends resistance to sublimation. This may furthermore suggest how the timing of snowfall relates to its persistence at this site, and ultimately the yield of spring runoff.

Lastly, it is evident from this data set that use of a single point measurement to calibrate the CRS probe will be unreliable given a heterogeneous snow pack. To counter heterogeneity, the collection of numerous points, for example as done by Sigouin and Si (2016), is highly recommended. Strategies for field calibrating the CRS sensor continue to evolve, so one should consult the latest literature. Clearly observations of the wide area snow pack, when combined with a high fidelity point measurements (e.g. Hydroinnova Snow Fox, snow pillow or snow scale), offer substantially more information than either measurement alone. Currently the Hydroinnova CRS is one of few products that can provide information on SWE at the scale of tens of hectares.

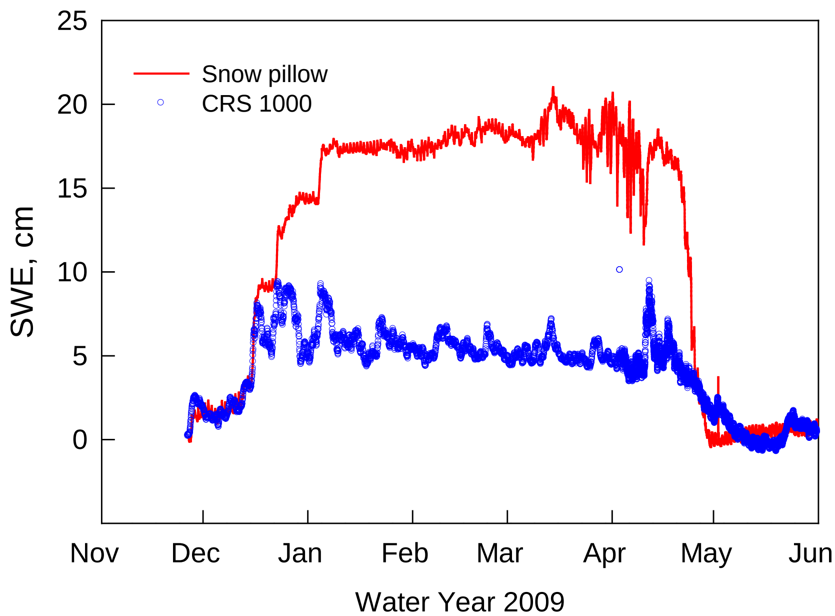

Figure 6. SWE measured in the Jémez Mountains with a snow pillow and a CRS probe using the CRS calibration from water year 2010.

Figure 7. Field site in the Jémez Mountains showing patchy snow coverage on April 3, 2009

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8. Summary and recommendations

In this paper I developed a function for converting neutron counts from a CRS probe to SWE. The function requires that the counting rate first be corrected, and that probe is calibrated for soil moisture, or that the soil moisture conditions are otherwise known. The function has two key parameters: Λ, which is an attenuation length, and N_{wat}, which is counting rate over deep water (>30 cm). Through Monte Carlo simulations Λ was found to have a value of 4.8 cm, and N_{wet}/N_0 was found to be 0.24, where N_0 is the counting rate over dry land. The function implies a rapidly diminishing sensitivity beyond 10 cm of SWE. Field data from a snow pillow were used to validate the function by showing that it gives a reasonable curvature, and implies a fairly reasonable water content of ~0.25 kg kg^{-1} for the study site. Lastly, I demonstrate that CRS data can still be valuable, even when the SWE for the site (in this case measured by a snow pillow) locally exceeds the nominal limits of the technique.

There is certainly room for improvements to this technique. One area for investigation is in dealing with a variable N_θ. A reasonable approximation might be to assume that N_θ transitions to a constant wet value in spring, due to the melting of snow. Another improvement might be to recast the calibration formula in terms of the depth of available water, combining storage in the snowpack with the storage in the shallow subsurface. Furthermore, more work needs to be done in distinguishing early season rain events from snow events, preferably from neutron data alone, for example by utilizing the bare channel in addition to the moderated channel.

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Revision History